QUANTUM FIELD THEORY IN CURVED SPACE-TIME AND THE EARLY UNIVERSE

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Abstract

New results on finite density of particle creation for nonconformal massive scalar particles in Friedmann Universe as well as new counterterms in dimensions higher than 5 are presented. Possible role of creation of superheavy particles for explaining observable density of visible and dark matter is discussed.

1 Introduction

Quantum field theory in curved space-time is the generalization of well developed theory in Minkowski space-time, so it has some features of the standard theory together with new ones due to the curvature. It was actively developed in the 70-ties of the last century (see our books [1, 2]) however some new results were obtained just recently and in this paper we'll concentrate on these new results as well as on some physical interpretation of the old ones.

One of the main results of the theory for the early Friedmann Universe was the calculation of finite stress-energy tensor for particle creation. Let us begin from some remarks on what is meant by particle creation in curved space-time? In spite of the absence of the standard definition of the particle as the representation of the Poincare group in curved space-time where Poincare group is not a group of motions, one still has physically clear idea of the particle. The particle is understood as the classical point like object moving along the geodesic of the curved space-time. This classical particle however is the quasiclassical approximation of some quantum object, so the main mathematical problem is to find the answer to the question: what is the Fock quantization of the field giving just this answer for particles in this approximation? The answer to this question was given by us in the 70-ties for conformal massive scalar particles and spinor particles in Friedmann space-time by use of the metrical Hamiltonian diagonalization method. Creation of particles in the early Universe from vacuum by the gravitational field means that for some small time close to singularity the stress-energy tensor has the geometrical form, expressed through some combinations of the Riemann tensor and its derivatives while for the time larger than the Compton one it has the form of the dust of pointlike particles with mass and spin defined by the corresponding Poincare group representation.

So our use of quantum field theory in curved space-time with its methods of regularization (dimensional regularization and Zeldovich-Starobinsky regularization) was totally justified by the obtained results. However still unclear was the situation with the minimally coupled scalar field where our method led to infinite results for the density of created particles, it is only

recently that Yu.V.Pavlov [3] could find the transformed Hamiltonian, diagonalization of which leads to finite results for this case. Also for higher dimensions it was found [4] that new counterterms different from those in 4-dimensional space-time appear in the Lagrangian.

2 Some remarks on the physical interpretation of terms in vacuum polarization dependent on mass

Our calculations of the vacuum expectation value of the stress-energy tensor of the quantized conformal massive scalar, spinor and vector fields [1, 2] led to the expressions different for strong and weak external gravitational field. By strong field one understands the field with the curvature much larger than the one defined by the mass of the particle. For strong gravitational field it is basically polarization of vacuum terms which becomes zero if the gravitation is zero, for weak gravitation it is defined by particles created previously, so that if gravitation becomes zero particles still exist. Vacuum polarization for strong gravitation consists of three terms: the first is due to the conformal anomaly and it does not depend on mass of the particle at all, the second is due to the Casimir effect and is present for the closed Friedmann space, the third is vacuum polarization dependent on mass of the particle which is for scalar conformal particles

$$<T_{ik}>_m = \frac{m^2}{144\pi^2}\, G_{ik} + \frac{m^4}{64\pi^2}\, g_{ik} \ln\left(\frac{\Re}{m^4}\right). \tag{1}$$

Here G_{ik} is the Einstein tensor, g_{ik} is the metrical tensor, $\Re$ is some geometrical term of the dimension of the fourth degree of mass. It is interesting that the first term is the illustration of the Sakharov's idea of gravitation as the vacuum polarization. If one takes the Planckean mass one has just the standard expression for the term in the Einstein equation. One can also think that gravitation is the manifestation of the vacuum polarization of all existing fields with different masses. However for weak gravitation for the time of evolution of the Friedmann Universe large than the Compton one defined by the mass there is no Sakharov term, it is compensated. It is just manifestation of the fact, known also in quantum electrodynamics that vacuum polarization is different in strong and weak external fields being asymptotics of some general expression. So the physical meaning of this term in strong field is finite change of the gravitational constant, so that the new effective gravitational constant is different from that in the weak field, being

$$\frac{1}{8\pi G_{eff}} = \frac{1}{8\pi G} + \frac{m^2}{144\pi^2}\,. \tag{2}$$

The second term on the right side (1) describes what is now called "quintessence" – cosmological constant, dependent on time. This "quintessence" is different for different stages of the evolution of the Friedmann Universe. We calculated it previously for the radiation dominated Universe, so that together with the Sakharov term one has

$$<T_{00}>_m = -\frac{m^2}{48\pi^2}\left(\frac{a'}{a^2}\right)^2 - \frac{m^4}{16\pi^2}\left\{\ln(ma) + C + \frac{1}{4} + \frac{1}{a^4}\int_0^\eta d\eta_1 \frac{da^2}{d\eta_1}\int_0^\eta d\eta_2 \frac{da^2}{d\eta_2}\ln|\eta_1 - \eta_2|\right\}. \tag{3}$$

Here a is the scale factor of the Friedmann space-time, η is the conformal time, $C = 0.577\ldots$ is the Euler constant.

3 The minimally coupled scalar field in homogeneous isotropic space

We consider a complex scalar field with Lagrangian

$$L(x) = \sqrt{|g|}\; [\, g^{ik} \partial_i \varphi^* \partial_k \varphi - (m^2 + \xi R)\, \varphi^* \varphi \,]\,, \tag{4}$$

where $g = \det\{g_{ik}\}$, R is the scalar curvature, ξ is a coupling to the curvature, which is equal to zero for the case of minimal coupling. The metric of N-dimensional homogeneous isotropic space-time is

$$ds^2 = a^2(\eta)\, (d\eta^2 - \gamma_{\alpha\beta} dx^\alpha dx^\beta)\,. \tag{5}$$

Here $\gamma_{\alpha\beta}$ is the metric of $(N-1)$-dimensional space of constant curvature $K = 0, \pm 1$.

Let us take a new Lagrangian different from the previous by the N-divergence $L^\Delta(x) = L(x) + \partial J^i / \partial x^i$, where (J^i) in coordinate system $(\eta, \mathbf{x})$ is given by

$$(J^i) = (\sqrt{\gamma}\, c\, \tilde{\varphi}^* \, \tilde{\varphi}\, (N-2)/2,\, 0,\, \dots\, , 0), \qquad \tilde{\varphi}(x) = a^{(N-2)/2}(\eta)\, \varphi(x)\,. \tag{6}$$

Here $c = a'/a, \quad \gamma = \det\{\gamma_{\alpha\beta}\}$. Lagrangian $L^\Delta(x)$ leads to the canonical Hamiltonian for the transformed field $\tilde{\varphi}$

$$H(\eta) = \int d^{N-1}x\, \sqrt{\gamma} \left\{ \tilde{\varphi}^{*\prime} \tilde{\varphi}' + \gamma^{\alpha\beta} \partial_\alpha \tilde{\varphi}^* \partial_\beta \tilde{\varphi} + \left[m^2 a^2 - \Delta\xi\, a^2 R + \left(\frac{N-2}{2} \right)^2 K \right] \tilde{\varphi}^* \tilde{\varphi} \right\}, \tag{7}$$

where $\Delta\xi = (N-2)/[4(N-1)] - \xi$.

For quantizing, we expand $\tilde{\varphi}(x)$ with respect to the complete set of motion equation solutions

$$\tilde{\varphi}(x) = \int d\mu(J) \left[\tilde{\varphi}^{(-)}_J\, a^{(-)}_J + \tilde{\varphi}^{(+)}_J\, a^{(+)}_J \right], \quad \tilde{\varphi}^{(+)}_J(x) = \frac{g_\lambda(\eta)\, \Phi^*_J(\mathbf{x})}{\sqrt{2}}, \quad \tilde{\varphi}^{(-)}_J(x) = \tilde{\varphi}^{(+)*}_J(x), \tag{8}$$

where $d\mu(J)$ is a measure in the space of eigenvalues of the Laplace-Beltrami operator Δ_{N-1} on $(N-1)$-dimensional space $\{x^\alpha\}$,

$$g''_\lambda(\eta) + \Omega^2(\eta) g_\lambda(\eta) = 0, \quad \Omega^2(\eta) = m^2 a^2 + \lambda^2 - \Delta\xi\, a^2 R, \quad \Delta_{N-1} \Phi_J = -\left(\lambda^2 - \frac{(N-2)^2}{4} K \right) \Phi_J. \tag{9}$$

The initial conditions corresponding to the diagonal form of Hamiltonian (7) in operators $\overset{*}{a}{}^{(\pm)}_J$ and $a^{(\pm)}_J$ in time η_0 are

$$g'_\lambda(\eta_0) = i\, \Omega(\eta_0)\, g_\lambda(\eta_0)\,, \quad |g_\lambda(\eta_0)| = 1/\sqrt{\Omega(\eta_0)}\,. \tag{10}$$

The density of the created pairs of particles [2] (for $N = 4$) is

$$n(\eta) = \frac{1}{2\pi^2 a^3(\eta)} \int d\lambda\, \lambda^2\, S_\lambda(\eta)\,, \tag{11}$$

where

$$S_\lambda(\eta) = \frac{1}{4\Omega} \left(|g_\lambda|^2 + \Omega^2 |g_\lambda|^2 \right) - \frac{1}{2}\,. \tag{12}$$

It can be shown [3], that $S_\lambda(\eta) \sim \lambda^{-6}, \;\; \lambda \to \infty, \;\;\; \forall\, \xi$. That is why the density of created particles of scalar field with arbitrary ξ in homogeneous isotropic 4-dimensional space-time is finite. So it is shown differently from [5], that even for minimal coupling the result is finite!

4 The renormalization for scalar field in N-dimensional quasi-Euclidean space-time

The vacuum expectation value of the stress-energy tensor of scalar field with Lagrangian (4) for vacuum corresponding to conditions (10) on N-dimensional space-time with $\gamma_{\alpha\beta} = \delta_{\alpha\beta}$ in (5) are

$$< 0 \,|\, T_{ik} |\, 0 >= \frac{B_N}{a^{N-2}} \int\limits_0^\infty d\lambda\, \lambda^{N-2} \tau_{ik}\,, \tag{13}$$

where $B_N = \left[2^{N-3} \pi^{(N-1)/2}\, \Gamma((N-1)/2) \right]^{-1}$, $\Gamma(z)$ is the gamma-function,

$$\tau_{00} = \Omega \Big(S + \frac{1}{2} \Big) + \Delta\xi\, (N-1) \left[c\, V + \Big(c' + (N-2)\, c^2 \Big) \frac{1}{\Omega} \left(S + \frac{1}{2} U + \frac{1}{2} \right) \right], \tag{14}$$

$$\begin{aligned} \tau_{\alpha\beta} \;&=\; \delta_{\alpha\beta} \left\{ \frac{\lambda^2}{(N-1)\,\Omega} \left(S + \frac{1}{2} \right) - \frac{\Omega^2 - \lambda^2}{2\,(N-1)\,\Omega}\, U - \right. \\ &-\; \left. \Delta\xi \left[(N-1)\, \frac{c'}{\Omega} \left(S + \frac{1}{2}\, U + \frac{1}{2} \right) + 2\,\Omega\, U - (N-1)\, c\, V \right] \right\}, \end{aligned} \tag{15}$$

$$S = \frac{|g'_\lambda|^2 + \Omega^2\, |g_\lambda|^2}{4\,\Omega} - \frac{1}{2}\,, \qquad U = \frac{\Omega^2\, |g_\lambda|^2 - |g'_\lambda|^2}{2\,\Omega} \qquad V = -\frac{d(g^*_\lambda g_\lambda)}{2\, d\eta}\,. \tag{16}$$

The vacuum expectation (13) has $[N/2]+1$ different types of divergences ($[b]$ denotes the integer part of b): $\sim \lambda^N,\ \lambda^{N-2}, \ldots, \ln\lambda$ if N is even, and $\sim \lambda^N,\ \lambda^{N-2}, \ldots, \lambda$ if N is odd. For renormalization we are using generalization for N-dimensional case [4] of n-wave procedure [6]

$$< 0 \,|\, T_{ik} |\, 0 >_{ren}= \frac{B_N}{a^{N-2}} \int\limits_0^\infty d\lambda\, \lambda^{N-2} \left[\tau_{ik} - \sum_{l=0}^{[N/2]} \tau_{ik}[l] \right], \tag{17}$$

$$\tau_{ik}[l] = \frac{1}{l!} \lim_{n\to\infty} \frac{\partial^l}{\partial (n^{-2})^l} \left(\frac{1}{n}\, \tau_{ik}(n\lambda, nm) \right). \tag{18}$$

The geometrical structure of counterterms has been determined with help of the dimensional regularization. It can be shown [4], that in the case $N = 4, 5$ all counterterms correspond to renormalization of constants in the bare gravitational Lagrangian of form

$$L_{gr,\,0} = \sqrt{|g|} \left[\frac{1}{16\pi G_0}\, (R - 2\Lambda_0) + \alpha_0 \left(R^{ik} R_{ik} - \frac{N\, R^2}{4(N-1)} \right) + \beta_0\, R^2 \right], \tag{19}$$

where R_{ik} is Ricci tensor. Constant α_0 has infinite renormalization under $N = 4 - \varepsilon,\ \varepsilon \to 0$.

New counterterms appear for $N \geq 6$ in comparison with a cases $N = 4, 5$, for example

$$\tau_{00}[3] = \omega\, S_6, \qquad \tau_{\alpha\beta}[3] = \frac{\delta_{\alpha\beta}}{(N-1)\,\omega} \left[\lambda^2\, S_6 - \frac{m^2 a^2}{2}\, U_6 \right], \tag{20}$$

where it's suggested that $\Delta\xi = 0$,

$$S_6 = \frac{5}{32}\, W^6 - \frac{5}{8}\, W^2 (DW)^2 - \frac{5}{4}\, W^3 D^2 W - \frac{1}{2} DW\, D^3 W + \frac{1}{2} W\, D^4 W + \frac{1}{4} \Big(D^2 W \Big)^2, \tag{21}$$

$$U_6 = \frac{15}{8}\, W^4 DW - \frac{5}{2}\, (DW)^3 - 10\, W\, DW\, D^2 W - \frac{5}{2}\, W^2 D^3 W + D^5 W\,, \tag{22}$$

$\omega = (m^2 a^2 + \lambda^2)^{1/2}$, $W = \omega' / (2\omega^2)$, $D = (2\omega)^{-1} (d\, .../d\eta)$. For $N = 6, 7$ from dimensional analysis of counterterms we can suppose that

$$\Delta L_{gr,\,0} = \sqrt{|g|} \left[\gamma_0 R^3 + \nu_0 R R^k_i R^i_k + \zeta_0 R^l_i R^k_l R^i_k + \rho_0 \nabla^l R\, \nabla_l R + \ldots \right]. \tag{23}$$

5 Superheavy particles in Friedmann cosmology and the dark matter problem

It is well known [2, 7] that creation of superheavy particles with the mass of the order of Grand Unification $M_X \approx 10^{14} - 10^{15}$ Gev with consequent decay on quarks and leptons with baryon charge and CP – nonconservation is sufficient for explanation of the observable baryonic charge of the Universe. Recently in papers [8, 9] the possibility of explanation of experimental facts on observation of cosmic ray particles with the energy higher than the Greizen-Zatsepin-Kuzmin limit was discussed. The proposal is to consider the decay of superheavy particles with the mass of the order M_X. One can even consider the hypothesis that all dark matter consists of neutral X – particles with very low density. So the problem is in the numerical estimate of the parameters of the effective Hamiltonian similar to the theory of K^0-mesons leading to the observable data. Short living X^0 – bosons decay on quarks and leptons in time close to singularity, long living X^0 – bosons exist today as the dark matter. Here we shall give this estimate.

Particles are created by gravitation at the Compton time $t \sim M^{-1}$ and for $t \gg M^{-1}$ one has nonrelativistic gas of created particles with the energy density calculated for the radiation dominated Friedmann model $a(t) = a_0\, t^{1/2}$: [2]

$$\varepsilon^{(0)} = 2b^{(0)}\, M(M/t)^{3/2}\,, \tag{24}$$

where $b^{(0)} \approx 5.3 \cdot 10^{-4}$. Total number of created particles in the Lagrange volume is

$$N = n^{(0)}(t)\, a^3(t) = b^{(0)}\, M^{3/2}\, a_0^3\,. \tag{25}$$

In spite of the cosmological order of the number of created X – particles ($N \sim 10^{80}$ for $M_X \sim 10^{15}$ Gev [2]) their back reaction on the background metric is small. [2] However for $t \gg M^{-1}$ there is an era of going from the radiation dominated model to the dust model of superheavy particles for $\varepsilon_{bground} \ \approx\ \varepsilon^{(0)}$,

$$t_X \approx \left(\frac{3}{64\pi\, b^{(0)}}\right)^2 \left(\frac{M_{Pl}}{M_X}\right)^4 \frac{1}{M_X}\,. \tag{26}$$

If $M_X \sim 10^{14}$ Gev, $t_X \sim 10^{-15}$ sec, if $M_X \sim 10^{13}$ Gev — $t_X \sim 10^{-10}$ sec. So the life time of short living X – mesons must be smaller then t_X. It is evident that if all created X – particles were stable, than the closed Friedmann model could quickly collapse, while all other models are strongly different from the observable Universe. Let us define d – the permitted part of long living X – mesons — from the condition: on the moment of recombination t_{rec} in the observable Universe one has

$$d\,\varepsilon_X(t_{rec}) = \varepsilon_{crit}(t_{rec})\,, \qquad d = \frac{3}{64\pi\, b^{(0)}} \left(\frac{M_{Pl}}{M_X}\right)^2 \frac{1}{\sqrt{M_X\, t_{rec}}}\,. \tag{27}$$

For $M_X = 10^{13} - 10^{14}$ Gev one has $d \approx 10^{-12} - 10^{-14}$. Using the estimate for the velocity of change of the concentration of long living superheavy particles [10] $|\dot{n}_x| \sim 10^{-42}\,\mathrm{cm}^{-3}\,\mathrm{sec}^{-1}$, and taking the life time τ_l of long living particles as $2 \cdot 10^{22}$ sec, we obtain concentration $n_X \approx 2 \cdot 10^{-20}\,\mathrm{cm}^{-3}$ at the modern epoch, corresponding to the critical density for $M_X = 10^{14}$ Gev.

Now let us construct the toy model which can give: a) short living X – mesons decay in time $\tau_q < 10^{-15}$ sec, (more wishful is $\tau_q \sim 10^{-38} - 10^{-35}$ sec), long living mesons decay with

$\tau_l > t_U \approx 10^{18}\,\mathrm{sec}$ (t_U – age of the Universe), b) one has small $d \sim 10^{-14} - 10^{-12}$ part of long living X – mesons, forming the dark matter.

Baryon charge nonconservation with CP – nonconservation in full analogy with the K^0 – meson theory with nonconserved hypercharge and CP – nonconservation leads to the effective Hamiltonian of the decaying $X, \bar{X}$ – mesons with nonhermitean matrix.

The matrix of the effective Hamiltonian is in standard notations

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}. \tag{28}$$

Let $H_{11} = H_{22}$ (due to CPT-invariance). Denote $\varepsilon = (\sqrt{H_{12}} - \sqrt{H_{21}}\,)\,/\,(\sqrt{H_{12}} + \sqrt{H_{21}}\,)$. The eigenvalues $\lambda_{1,2}$ and eigenvectors $|\,\Psi_{1,2}>$ of matrix H are

$$\lambda_{1,2} = H_{11} \pm \frac{H_{12}+H_{21}}{2}\,\frac{1-\varepsilon^2}{1+\varepsilon^2}\,, \tag{29}$$

$$|\,\Psi_{1,2}>= \frac{1}{\sqrt{2\,(1+|\varepsilon|^2)}}\Bigl[(1+\varepsilon)\,|\,1> \pm (1-\varepsilon)\,|\,2>\Bigr]. \tag{30}$$

In particular

$$H = \begin{pmatrix} E - \frac{i}{4}\left(\tau_q^{-1} + \tau_l^{-1}\right) & \frac{1+\varepsilon}{1-\varepsilon}\left[A - \frac{i}{4}\left(\tau_q^{-1} - \tau_l^{-1}\right)\right] \\ \frac{1-\varepsilon}{1+\varepsilon}\left[A - \frac{i}{4}\left(\tau_q^{-1} - \tau_l^{-1}\right)\right] & E - \frac{i}{4}\left(\tau_q^{-1} + \tau_l^{-1}\right) \end{pmatrix}. \tag{31}$$

Then the state $|\,\Psi_1>$ describes short living particles with the life time τ_q and mass $E+A$. The state $|\,\Psi_2>$ is the state of long living particles with life time τ_l and mass $E-A$. Here A is the arbitrary parameter $-E < A < E$ and it can be zero, $E = M_X$.

If $d = 1 - |<\Psi_1\,|\,\Psi_2>|^2 = 1 - |2\,\mathrm{Re}\,\varepsilon/(1+|\varepsilon|^2)|^2$ is the relative part of long living particles and ε is real, then $\varepsilon = (1-\sqrt{d}\,)/\sqrt{1-d}$ ($\varepsilon \sim 1 - 10^{-7}$ for $M_X \sim 10^{14}\,\mathrm{Gev}$). So one has a typical example of "fine tuning" in order to obtain the desired result.

Taking $\tau_q = 10^{-35}$ sec and $\tau_l = 2 \cdot 10^{22}$ sec, one obtains that for the Hermitean part $(H+H^+)/2$ of H nondiagonal CP-noninvariant term equal to $i\,(\tau_q^{-1} - \tau_l^{-1})\,\varepsilon/(1-\varepsilon^2)/2$ is of the order of Planckean mass 10^{19} Gev.

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